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FACTOR BIAS IN INNOVATIONS

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The literature on the Averch-Johnson (A-J) effect focuses on the distortion that is introduced into the capital-labor ratio employed by a regulated firm as the result of rate of return regulation.[1] This paper considers a related question, namely that of the effect of rate of return regulation on the mix of labor and capital-augmenting innovations produced and employed by a regulated firm. The regulated firm is assumed to possess "in house" capabilities for producing such innovations, and it is the responsiveness of its innovative activities to regulation that is the central theme of this paper. But the mix of innovative activities also affects the firm's choice of a capital/labor ratio in producing its final product. This more conventional notion of factor bias is also explored in the paper.

We have considered in detail two specifications of the regulatory scheme, one in which all of the firm's capital is included in its rate base, and a second case in which only the capital employed in producing the final product of the firm is eligible for inclusion in the rate base. The first case is obviously unrealistic, but it turns out that the second can be regarded as a special case of the first, so it is convenient to study it before turning to the more realistic

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1. See Baumol, W. and Klevorick (1970).

case in which the capital employed in producing innovations is excluded from the rate base. In addition, it is of some interest to see how an alternative specification of the rate base affects the pattern of bias in the firm's activities.

The general conclusions that are presented in this paper are the following. The intuitive notion that rate of return regulation will tend to bias innovative activities in the direction of higher levels of labor-augmenting to capital-augmenting innovations must be qualified to take into account the elasticity of substitution between augmented capital and labor in producing the firm's final product, and the dependence of the output of innovations on the levels of technical progress already achieved by the firm. When all of a firm's capital enters the rate base, the relative capital intensities of the firm's innovative activities also play a role in determining the direction of bias. Generally speaking, when capital and labor are relatively poor substitutes for one another in producing the final product, and when the output of innovations is relatively insensitive to the state of progress already achieved, then the expected distortion in the production of innovations occurs. But the expected distortion will be reversed for a range of values of the elasticity of substitution, this range depending on the sensitivity of innovative outputs to the levels already achieved. Interestingly, the distortion in the capital/labor ratio employed in producing the final product occurs independently of the size of the elasticity of substitution, so that the traditional A-J distortion is preserved even when innovations are included in the regulated firm's activities.

The conclusions that are derived in the paper all relate to steady states for the firm along a balanced growth path where factor prices and capital/labor ratios in all the firm's activities are constant over time. This provides a rough comparability with the usual static versions of the A-J model.

The Model

The model employed is of a monopolistic firm which produces a final product that is sold in a regulated market. The firm also has "in house" capabilities to produce innovations that augment the labor and capital employed in producing the final product. L and K refer to labor and capital respectively, while w and r are the wage rate and the rental rate. Y is output, with $p(Y)$ the inverse demand function facing the firm. $A(t)$ is an index of capital productivity at time t, while $B(t)$ is the corresponding index applied to labor. The allowed rate of return per dollar of capital is s. A distinction is drawn among the uses to which capital and labor are put, with K_1 denoting capital employed in producing the final product; K_2, capital employed in producing capital augmenting innovations; and K_3, capital used to produce labor-augmenting innovations. A similar notation is employed with respect to labor.

The Case Where All of the Firm's Capital Appears in the Rate Base

We consider first the case where the rate base of the firm is K, the total amount of capital employed by the firm.

The firm's problem may be formulated as follows:

$$\max \int_{\sigma}^{\infty} [p(Y)F(AK_1, BL_1) - wL - rK]e^{-\delta t}dt$$

subject to

$$\dot{A} = \phi(A, K_2, L_2)$$

$$\dot{B} = \psi(B, K_3, L_3)$$

$$p(Y)F - wL \leq sK$$

$$A(o) = A_o, B(o) = B_o$$

where

$$L = L_1 + L_2 + L_3, \quad K = K_1 + K_2 + K_3, \quad Y = F(AK_1, BL_1).$$

Let

$$H = [pF - wL - rK]e^{-\delta t} + \lambda_1\phi + \lambda_2\psi + \lambda_3[sK - pF + wL]$$

First order conditions are given by:

$$(1) \quad \frac{\partial H}{\partial K_1} = [(MR)F_1A - r]e^{-\delta t} + \lambda_3[s - (MR)F_1A] = 0$$

$$(2) \quad \frac{\partial H}{\partial L_1} = [(MR)F_2B - w](e^{-\delta t} - \lambda_3) = 0$$

$$(3) \quad \frac{\partial H}{\partial K_2} = -re^{-\delta t} + \lambda_1\phi_K + \lambda_3 s = 0$$

$$(4) \quad \frac{\partial H}{\partial L_2} = -we^{-\delta t} + \lambda_1\phi_L + \lambda_3 w = 0$$

$$(5) \quad \frac{\partial H}{\partial K_3} = -re^{-\delta t} + \lambda_2\psi_K + \lambda_3 s = 0$$

$$(6) \quad \frac{\partial H}{\partial L_3} = -we^{-\delta t} + \lambda_2\psi_L + \lambda_3 w = 0$$

$$(7) \quad \dot{\lambda}_1 = -\frac{\partial H}{\partial A} = -[(MR)F_1K_1(e^{-\delta t} - \lambda_3) + \lambda_1\phi_A]$$

$$(8) \quad \dot{\lambda}_2 = -\frac{\partial H}{\partial B} = -[(MR)F_2L_1(e^{-\delta t} - \lambda_3) + \lambda_2\psi_B]$$

$$(9) \quad \dot{A} = \phi$$

$$(10) \quad \dot{B} = \psi$$

with transversality conditions $\lim_{t \to \infty} \lambda_1 = 0$, $\lim_{t \to \infty} \lambda_2 = 0$.

$$\left(MR = p + Y\frac{dp}{dY}, \quad F_1 = \frac{\partial F}{\partial(AK_1)}, \quad F_2 = \frac{\partial F}{\partial(BL_1)}\right)$$

We will work with the special case in which

$$\lim_{Y \to 0} p(Y) = +\infty, \quad \lim_{Y \to \infty} p(Y) = 0, \quad \frac{dp}{dY} < 0, \quad Y \geq 0.$$

Further it is assumed that F, ϕ and ψ are well behaved neoclassical production functions. In particular, F is homgeneous of degree $k > 1$ in AK_1, BL_1, while ϕ and ψ are homogeneous of degree one in K_2, L_2, and K_3, L_3 respectively.

Thus

$$F(AK_1, BL_1) = (BL_1)^k F(v_1, 1) = (BL_1)^k f(v_1)$$

where

$$v_1 = \frac{AK_1}{BL_1}, \quad f(v_1) = F(v_1, 1)$$

Further,

$$f'(v_1) > 0, \quad f''(v_1) < 0 \text{ for } v_1 \geq 0$$

with

$$\lim_{v_1 \to 0} f'(v_1) = +\infty, \quad \lim_{v_1 \to \infty} f'(v_1) = 0.$$

Similarly, assume that

$$\phi(A, K_2, L_2) = \alpha(A)G(K_2, L_2) \equiv \alpha(A)L_2 g(v_2),$$

while

$$\psi(B, K_3, L_3) = \gamma(B)H(K_3, L_3) \equiv \gamma(B)L_3 h(v_3),$$

where

$$v_2 = \frac{K_2}{L_2}, \quad v_3 = \frac{K_3}{L_3}.$$

g and h are assumed to possess the same neoclassical properties as f.[2]

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2. Separability of ϕ and ψ is assumed in order to simplify the analysis of the dependence of $\dot{A}$ and $\dot{B}$ on the levels of A and B already achieved.

Finally, let $\mu_1 = \lambda_1 e^{\delta t}$, $\mu_2 = \lambda_2 e^{\delta t}$, $\mu_3 = \lambda_3 e^{\delta t}$. Then the first order conditions (1)-(10) can be rewritten as follows.

(1′) $(MR)(BL_1)^{k-1} f' A(1-\mu_3) + \mu_3 s - r = 0$

(2′) $[(MR)(BL_1)^{k-1}(f-v_1 f')B - w](1-\mu_3) = 0$

(3′) $-r + \mu_1 \alpha g' + \mu_3 s = 0$

(4′) $\mu_1 \alpha(g-v_2 g') - w(1-\mu_3) = 0$

(5′) $-r + \mu_2 \gamma h' + \mu_3 s = 0$

(6′) $\mu_2 \gamma(h-v_3 h') - w(1-\mu_3) = 0$

(7′) $\dot{\mu}_1 = \delta\mu_1 - (MR)(BL_1)^{k-1} f' K_1 (1-\mu_3) - \mu_1 \alpha' L_2 g$

(8′) $\dot{\mu}_2 = \delta\mu_2 - (MR)(BL_1)^{k-1}(f-v_1 f')L_1(1-\mu_3) - \mu_2 \gamma' L_3 h$

(9′) $\dot{A} = \alpha L_2 g$

(10′) $\dot{B} = \gamma L_3 h$

with transversality conditions $\lim_{t\to\infty} \mu_1 e^{-\delta t} = 0$, $\lim_{t\to\infty} \mu_2 e^{-\delta t} = 0$.

As in the traditional A-J literature, we assume that the rate of return constraint is strictly binding at each point in time in the sense that s is strictly less than the rate of return for the unregulated firm. Hence $\mu_3 > 0$ for all t. Further, concavity of H and R (= pF) implies $\mu_3 < 1$.[3]

Then from (1′)-(6′) we obtain

$$\frac{w(1-\mu_3)}{r-\mu_3 s} = \left(\frac{f-v_1 f'}{f'}\right)\frac{B}{A} = \frac{g-v_2 g'}{g'} = \frac{h-v_3 h'}{h'}.$$

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3. Note that (2) can be written as $\frac{\partial H}{\partial L_1} = (R_L - w)(e^{-\delta t} - \lambda_3)$ so that $\frac{\partial^2 H}{\partial L_1^2} = R_{LL}(e^{-\delta t} - \lambda_3) < 0$, $R_{LL} < 0$ implies $\mu_3 < 1$. Clearly, conditions weaker than concavity of H can also be used to establish that $\mu_3 < 1$.

Alternatively,

$$\frac{A}{B}\left(\frac{f'}{f-v_1 f'}\right) = \frac{g'}{g-v_2 g'} = \frac{h'}{h-v_3 h'} = \frac{r}{w} - \frac{\mu_3(s-r)}{(1-\mu_3)w} < \frac{r}{w}.$$

Since

$$\frac{d}{dv_2}\left(\frac{g'}{g-v_2 g'}\right) = \frac{g g''}{(g-v_2 g')^2} < 0,$$

with

$$\frac{d}{dv_3}\left(\frac{h'}{h-v_3 h'}\right) = \frac{h h''}{h-v_3 h')^2} < 0; \quad \frac{d}{dv_1}\left(\frac{f'}{f-v_1 f'}\right) = \frac{f f''}{(f-v_1 f')^2} < 0,$$

it follows that for any given levels of $A, \dot{A}, B, \dot{B}, Y$, the usual A-J distortion occurs, with capital/labor ratios in innovative activities and the augmented capital/augmented labor ratio in producing the final product exhibiting over-capitalization, given that $0 < \mu_3 < 1$, $r < s < r_{max}$, where r_{max} is the rate of return for the firm in the absence of regulation.

But we are interested instead in the responsiveness of the level of innovative activities of the firm to the existence of a rate of return constraint. We will examine this in the simplest possible setting, namely one in which the capital/labor ratios in all of the firm's activities are assumed to be constant over time; thus we are explicitly restricting our analysis to a steady state balanced growth path of the firm. Assuming s, w, r are constant over time, then the existence of a steady state implies that $\dot{v}_2 = \dot{v}_3 = 0$, together with $\dot{q}_1 = 0$, where $q_1 = \frac{K_1}{L_1}$. Note that $q_1 = \frac{B}{A} v_1$ so that $\dot{q}_1 = 0$ implies that

$$\frac{B}{A}\dot{v}_1 + v_1\left(\frac{A\dot{B} - B\dot{A}}{A^2}\right) = 0$$

thus

$$\frac{B}{A}\left[\dot{v}_1 + v_1\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right)\right] = 0 \text{ or } \dot{v}_1 = v_1\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right).$$

Since

$$\frac{w(1-\mu_3)}{r-\mu_3 s} = \left(\frac{f-v_1 f'}{f'}\right)\frac{B}{A} = \frac{g-v_2 g'}{g'} = \frac{h-v_3 h'}{h'}$$

are identities in t, it follows that

$$\frac{w(s-r)\dot{\mu}_3}{(r-\mu_3 s)^2} = \frac{-gg''}{(g')^2}\dot{v}_2 = \frac{-hh''}{(h')^2}\dot{v}_3.$$

Thus $\dot{v}_2 = \dot{v}_3 = 0$ implies that $\dot{\mu}_3 = 0$. Further we have

$$\frac{-ff''}{(f')^2}\frac{B}{A}\dot{v}_1 + \left(\frac{f-v_1 f'}{f'}\right)\frac{A\dot{B}-B\dot{A}}{A^2} = 0$$

along a balanced growth path.

Thus

$$\frac{\dot{v}_1}{v_1} = \sigma_f\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)$$

where σ_f is the elasticity of substitution between augmented capital and labor in producing the final product of the firm.

Then

$$\frac{\dot{q}_1}{q_1} = (\sigma_f - 1)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)$$

so that the condition $\dot{q}_1 = 0$ implies either that the production function F is Cobb-Douglas or that the percentage rates of increase in capital and labor augmentation are equal.

These conclusions are the usual ones found in the factor bias literature (see Samuelson (1965)). But there is an additional condition on the rates of technical progress that follows from profit maximization. Since (3′) is an identity in t, then along a balanced growth path with w, r, s constant we have

$$\dot{\mu}_1 \alpha g' + \mu_1 g' \alpha' \dot{A} = 0$$

Thus

$$\dot{\mu}_1 = \frac{-\mu_1 \alpha' \dot{A}}{\alpha}.$$

Combining this with (7′) we have

$$(MR)f'K_1(BL_1)^{k-1}(1-\mu_3) = \delta\mu_1 - \mu_1\alpha' L_2 g + \frac{\mu_1 \alpha' \dot{A}}{\alpha}$$

Since $\dot{A} = \alpha L_2 g$, it follows that

$$(MR)f'(K_1)(BL_1)^{k-1}(1-\mu_3) = \delta\mu_1.$$

Similarly, differentiating (5′) with respect to t we obtain

$$\dot{\mu}_2 \gamma h' + \mu_2 h' \gamma' \dot{B} = 0,$$

hence

$$\dot{\mu}_2 = -\mu_2 \frac{\gamma'}{\gamma}\dot{B}.$$

Combining with (8′) we have

$$(MR)(f - v_1 f')L_1(BL_1)^{k-1}(1-\mu_3) = \delta\mu_2 + \mu_2\frac{\gamma'}{\gamma}\dot{B} - \mu_2\gamma' L_3 h$$

so that

$$(MR)(f - v_1 f')L_1(BL_1)^{k-1}(1-\mu_3) = \delta\mu_2.$$

Hence

$$\left(\frac{f'}{f - v_1 f'}\right) q_1 = \frac{\mu_1}{\mu_2} = \frac{\gamma h'}{\alpha g'}$$

from (3′) and (5′) so that

$$q_1 = \left(\frac{f - v_1 f'}{f'}\right) \frac{\gamma h'}{\alpha g'} = \frac{A}{B}\left(\frac{w(1-\mu_3)}{r - \mu_3 s}\right) \frac{\gamma h'}{\alpha g'} .$$

Thus

$$\frac{\dot{q}_1}{q_1} = \left(\frac{\dot{A}}{B} - \frac{\dot{B}}{B}\right) + \frac{\gamma' \dot{B}}{\gamma} - \frac{\alpha' \dot{A}}{\alpha} .$$

Along a balanced growth path, $\dot{q}_1 = 0$. Thus $\sigma_f \neq 1$ implies that along a balanced growth path, $\frac{\dot{A}}{A} = \frac{\dot{B}}{B}$ and $\frac{\gamma' \dot{B}}{\gamma} = \frac{\alpha' \dot{A}}{\alpha}$, which implies $\frac{\gamma' B}{\gamma} = \frac{\alpha' A}{\alpha}$. Hence $\frac{\gamma}{\alpha} = \frac{\gamma'}{\alpha'} \frac{B}{A}$. Thus along a balanced growth path with $\sigma_f \neq 1$,

$$q_1 = \left(\frac{w(1-\mu_3)}{r - \mu_3 s}\right) \frac{\gamma' h'}{\alpha' g'} .$$

Let $\omega^* = \frac{w(1-\mu_3)}{r - \mu_3 s}$, with $\omega = \frac{w}{r}$. Since $0 < \mu_3 < 1$, $0 < r < s$, thus $\omega^* > \omega$.

Comparative Dynamics

We will examine the effects of rate of return regulation on the innovative activities of the firm by considering the impact of an increase in ω^* on the steady state magnitudes characterizing the firm. It should perhaps be mentioned that $A, \dot{A}, B, \dot{B}$ are all time dependent,

even under steady state conditions. However, assuming $\sigma_f \neq 1$ implies that $\frac{\dot{A}}{A} = \frac{\dot{B}}{B}$ along a steady state balanced growth path, so that B/A is constant along such a path.

Note that

$$\omega^* = \left(\frac{f}{f'} - v_1\right)\frac{B}{A} .$$

Thus

$$\frac{A}{B}\frac{d(B/A)}{d\omega^*} = \frac{1}{\omega^*} - \left(\frac{1}{\sigma_f}\right)\frac{1}{v_1}\frac{dv_1}{d\omega^*} .$$

Further, since $v_1 = \frac{A}{B} q_1$,

$$\frac{1}{v_1}\frac{dv_1}{d\omega^*} = \frac{1}{q_1}\frac{dq_1}{d\omega^*} - \frac{A}{B}\frac{d(B/A)}{d\omega^*} ,$$

so that

$$\frac{A}{B}\frac{d(B/A)}{d\omega^*} = \frac{\sigma_f}{\sigma_f - 1}\left\{\frac{1}{\omega^*} - \left(\frac{1}{\sigma_f}\right)\frac{1}{q_1}\frac{dq_1}{d\omega^*}\right\} .$$

Morever,

$$q_1 = \frac{A}{B}\omega^* \frac{\gamma}{\alpha}\frac{h'}{g'} .$$

$\therefore$

$$\frac{1}{q_1}\frac{dq_1}{d\omega^*} = \frac{1}{\omega^*} + \frac{B}{A}\frac{d(A/B)}{d\omega^*} + \frac{\alpha}{\gamma}\frac{d(\gamma/\alpha)}{d\omega^*} + \frac{g'}{h'}\frac{d(h'/g')}{d\omega^*} .$$

From the first order conditions we have

$$\omega^* = \frac{h}{h'} - v_3 = \frac{g}{g'} - v_2$$

so that

$$\frac{dv_3}{d\omega^*} = \frac{-(h')^2}{hh''}, \quad \frac{dv_2}{d\omega^*} = \frac{-(g')^2}{gg''}.$$

Further, for any t,

$$\frac{d(B/A)}{d\omega^*} = \frac{B}{A}\left(\frac{dB(t)}{d\omega^*}\Big/B - \frac{dA(t)}{d\omega^*}\Big/A\right)$$

while

$$\frac{d(\gamma/\alpha)}{d\omega^*} = \frac{\gamma}{\alpha}\left(\frac{\gamma'}{\gamma}\frac{dB(t)}{d\omega^*} - \frac{\alpha'}{\alpha}\frac{dA(t)}{d\omega^*}\right)$$

$$= \left(\frac{\gamma}{\alpha}\right)\left(\frac{\gamma'}{\gamma}\right)\frac{Ad(B/A)}{d\omega^*} + \frac{\gamma}{\alpha}\frac{dA(t)}{d\omega^*}\left[\frac{\gamma'}{\gamma}\frac{B}{A} - \frac{\alpha'}{\alpha}\right].$$

Since $\frac{\gamma' B}{\gamma} = \frac{\alpha' A}{\alpha}$ along a balanced growth path,

$$\frac{d(\gamma/\alpha)}{d\omega^*} = \frac{\gamma' A}{\alpha}\frac{d(B/A)}{d\omega^*} = \frac{A}{B}\frac{d(B/A)}{d\omega^*}\left(\frac{\gamma' B}{\alpha}\right).$$

Hence

$$\frac{1}{q_1}\frac{dq_1}{d\omega^*} = \frac{1}{\omega^*} - \frac{A}{B}\frac{d(B/A)}{d\omega^*} + \frac{\gamma' B}{\gamma}\frac{d(B/A)}{d\omega^*}\left(\frac{A}{B}\right)$$

$$+ \frac{1}{\omega^* + v_2} - \frac{1}{\omega^* + v_3}.$$

Solving, we obtain

$$\frac{1}{(B/A)}\frac{d(B/A)}{d\omega^*}\left(1 + \frac{\gamma' B/\gamma - 1}{\sigma_f - 1}\right) = \frac{\sigma_f}{\sigma_f - 1}\left(\frac{1}{\omega^*}\right)$$

$$- \frac{1}{\sigma_f - 1}\left(\frac{1}{\omega^*} + \frac{1}{\omega^* + v_2} - \frac{1}{\omega^* + v_3}\right).$$

∴

$$\frac{\omega^*}{(B/A)}\frac{d(B/A)}{d\omega^*} = \frac{1}{(\sigma_f - 1) + \left(\frac{\gamma' B}{\gamma} - 1\right)}\left\{(\sigma_f - 1) + \frac{\omega^*(v_2 - v_3)}{(\omega^* + v_2)(\omega^* + v_3)}\right\}$$

and

$$\frac{\omega^*}{q_1}\frac{dq_1}{d\omega^*} = \frac{\omega^*(v_2 - v_3)}{(\omega^* + v_2)(\omega^* + v_3)}\left(\frac{-(\sigma_f - 1)}{(\sigma_f - 1) + \left(\frac{\gamma' B}{\gamma} - 1\right)}\right)$$

$$+ \frac{(\sigma_1 - 1) + \sigma_f(\gamma' B/\gamma - 1)}{(\sigma_f - 1) + \left(\frac{\gamma' B}{\gamma} - 1\right)}.$$

These are admittedly rather complicated expressions and deserve some interpretation. First, note that $\dot{B} = \gamma(B)L_3 h$, so that if K_3, L_3 are held constant, $\frac{B}{\dot{B}}\frac{\partial \dot{B}}{\partial B} = \frac{\gamma' B}{\gamma}$. Thus $\frac{\gamma' B}{\gamma}\left(= \frac{\alpha' A}{\alpha}\right)$ is the elasticity of the output of labor augmenting innovations with respect to the level of such augmentation already achieved, for given inputs of capital and labor into the innovative process.

We assume the usual S-shaped curve of technical progress as in figure 1, with the corresponding $\gamma(B)$ function in figure 2.

In figure 1, the slope of the B(t) function increases to the inflexion point t^* and falls thereafter. The corresponding $\gamma(B)$ function is concave, as shown in figure 2. But $\gamma(B)$ concave implies that $\gamma' < \frac{\gamma}{B}$, that is, the slope of γ at any point B is always less than the slope of a line drawn from the origin to the curve $\gamma(B)$. Hence $\frac{\gamma' B}{\gamma} < 1$.

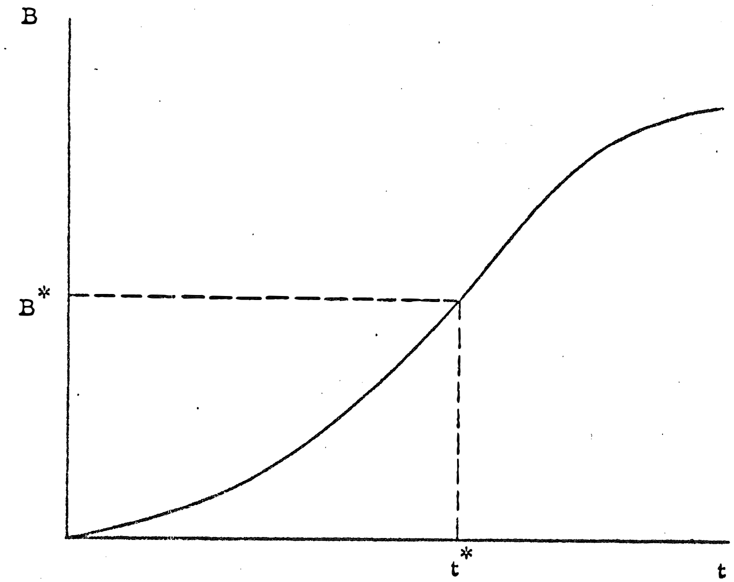

Figure 1. The B(t) function

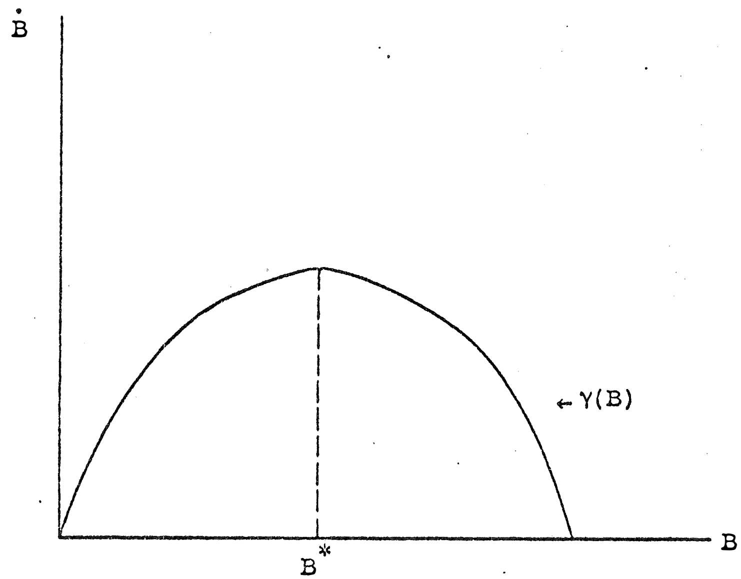

Figure 2. The γ(B) function

$\frac{d(B/A)}{d\omega^*}$ and $\frac{dq_1}{d\omega^*}$ depend upon three properties of the firm's technology:

(1) the elasticity of substitution between augmented labor and augmented capital in producing the final product of the firm;

(2) the relative capital intensities v_2 and v_3 in the production of capital augmenting and labor augmenting technical progress;

(3) the elasticity of output of innovations with respect to the level of progress already achieved.

An increase in ω^* (say due to the imposition of rate of return regulation) tends by itself to bias the hiring of inputs so as to increase the capital/labor ratio, and tends to bias innovations toward labor augmenting (B) at the expense of capital augmenting (A) innovations. But to the extent that capital augmenting innovations are more capital intensive than labor augmenting innovations, the tendency towards this bias is offset; and the ease of substitution between augmented labor and capital in the final product acts to influence such a bias as well. Finally, the higher the elasticity of output of innovations with respect to the level already achieved, the less will the bias show up in the patterns of hirings and innovative activities of the firm.

In particular, assume that $v_2 = v_3$ -- the capital intensities in both innovative activities are identical. Then $\frac{d(B/A)}{d\omega^*} > 0$ for σ_f sufficiently small; but the bias induced by regulation leads to $\frac{d(B/A)}{d\omega^*} < 0$ for $1 < \sigma_f < 2 - \frac{\gamma' B}{\gamma}$. In contrast $\frac{dq_1}{d\omega^*} > 0$ for all values of σ_f -- a reversal of the "expected" A-J bias can occur only because of differing capital intensities of the innovative processes.

The special case where $v_2 = v_3$ is of some interest, because it turns out that the "realistic" case where only K_1 appears in the rate base is equivalent (in terms of qualitative properties) to the case where $v_2 = v_3$.

The Case in Which Only K_1 Appears in the Rate Base

Assume that only capital employed in producing the final product of the firm appears in its rate base, while costs incurred in innovative activities are considered a deduction from revenue in calculating allowed profits. Using the notation above, the problem becomes

$$\max \int_0^\infty [\, p(Y)F(AK_1, BL_1) - wL - rK]\, e^{-\delta t}\, dt$$

subject to

$$\dot{A} = \phi(A, K_2, L_2)$$

$$\dot{B} = \psi(B, K_3, L_3)$$

$$p(Y)F - wL - r(K_2 + K_3) \leqq sK_1$$

In terms of output per unit of labor production functions, the first order conditions for this problem are the following.

(1^*) $(MR)\,(BL_1)^{k-1} f' A(1 - \mu_3^*) + \mu_3^* s - r = 0$

(2^*) $[(MR)\,(BL_1)^{k-1}(f - v_1 f')B - w]\,(1 - \mu_3^*) = 0$

(3^*) $\mu_1^* \alpha g' - r(1 - \mu_3^*) = 0$

(4^*) $\mu_1^* \alpha(g - v_2 g') - w(1 - \mu_3^*) = 0$

(5^*) $\mu_2^* \gamma h' - r(1 - \mu_3^*) = 0$

(6^*) $\mu_2^* \gamma(h - v_3 h') - w(1 - \mu_3^*) = 0$

(7^*) $\dot{\mu}_1^* = \delta\mu_1^* - (MR)\,(BL_1)^{k-1} f' K_1(1 - \mu_3^*) - \mu_1^* \alpha' L_2 g$

(8^*) $\dot{\mu}_2^* = \delta\mu_2^* - (MR)\,(BL_1)^{k-1}(f - v_1 f')L_1(1 - \mu_3^*) - \mu_2^* \gamma' L_3 h$

(9^*) $\dot{A} = \alpha L_2 g$

(10^*) $\dot{B} = \gamma L_3 h,$

together with

$$\lim_{t \to \infty} \mu_1^* e^{-\delta t} = 0, \quad \lim_{t \to \infty} \mu_2^* e^{-\delta t} = 0.$$

It will be noted that the conditions (1^*) - (10^*) are identical to $(1')$ - $(10')$ except that in (3^*) and (5^*), s is replaced by r. The expected results occur so far as factor distortion is concerned, along a balanced growth path. That is,

$$\omega = \frac{g - v_2 g'}{g'} = \frac{h - v_3 h'}{h'},$$

$$\frac{B}{A}\,\frac{f - v_1 f'}{f'} = \frac{\omega(1 - \mu_3^*)}{r - \mu_3^* s} \equiv \omega^* > \omega .$$

Hence capital and labor are employed in the proper cost minimizing proportions so far as innovative activities are concerned while there is a distortion in the augmented capital/augmented labor ratio in the final product.

If w, r, s are constant over time, we have

$$\dot{\omega} = 0 = \frac{-gg''}{(g')^2}\,\dot{v}_2 = \frac{-hh''}{(h')^2}\,\dot{v}_3,$$

so that v_2 and v_3 are constant over time for any given $\omega = \frac{w}{r}$.

In contrast to the case where all capital enters the firm's rate base, balanced growth ($\dot{v}_2 = \dot{v}_3 = \dot{q}_1 = 0$) does not necessarily imply that $\dot{\mu}_3^* = 0$. Instead, we will explicitly assume that along a balanced growth path, $\frac{\dot{A}}{A} = \frac{\dot{B}}{B}$, and $\dot{q}_1 = 0$. This implies that $\dot{\mu}_3^* = 0$.

We consider the impact on (B/A) assuming ω^* increases, with ω held fixed. Thus $\frac{\omega^*}{(B/A)} \frac{d(B/A)}{d\omega^*}$ measures the impact on the factor bias in innovations of rate of return constraint. Because ω is held fixed, v_2 and v_3 are not affected by the rate of return regulations. Using the same approach as in the previous section, we obtain

$$\frac{\omega^*}{(B/A)} \frac{d(B/A)}{d\omega^*} = \frac{\sigma_f - 1}{(\sigma_f - 1) + \left(\frac{\gamma' B}{\gamma} - 1\right)}$$

$$\frac{\omega^*}{q_1} \frac{dq_1}{d\omega^*} = \frac{(\sigma_f - 1) + \sigma_f(\gamma' B/\gamma - 1)}{(\sigma_f - 1) + \left(\frac{\gamma' B}{\gamma} - 1\right)}$$

Note that when only K_1 enters the rate base, $\frac{dv_2}{d\omega^*} = \frac{dv_3}{d\omega^*} = 0$ hence the terms involving $\frac{\omega^* (v_3 - v_2)}{(\omega^* + v_2)(\omega^* + v_3)}$ vanish from the corresponding expressions derived for the case where all capital enters the rate base. So far as the effect of the restriction of the rate base to K_1 on factor bias is concerned, dropping K_2 and K_3 from the rate base has an effect only if $v_2 \neq v_3$. It follows that with only K_1 in the rate base, $\frac{d(B/A)}{d\omega^*} > 0$ for small values of σ_f with $\frac{d(B/A)}{d\omega^*} < 0$ for $1 < \sigma_f < 2 - \frac{\gamma' B}{\gamma}$. $\frac{dq_1}{d\omega^*} > 0$ for all values of σ_f and $\frac{\gamma' B}{\gamma}$, so long as $\gamma(B)$ is concave.

Note that the A-J bias induced in q_1 by rate of return regulation could, in fact, be reduced or perhaps eliminated by including all of the firm's capital in its rate base, so long as

$v_2 > v_3$ and σ_f is sufficiently large (or $v_2 < v_3$ with σ_f sufficiently small). On the other hand, the factor bias in inputs used in innovative activities only appears if K_2 and K_3 are in the rate base.

Finally, it might be noted that the specification of the firm's decision problem in the case where only K_1 enters the rate base assumes that the regulating authority can, in fact, identify the costs of the firm's innovative activities and allows only such costs as a deduction from revenues. If the regulated firm instead is permitted to include in its allowable costs any purchases of innovative inputs from its wholely owned subsidiary, with prices chosen by the subsidiary in a profit maximizing fashion, then the firm can achieve a monopoly price-quantity position; rate of return regulation turns out to be ineffective.

Conclusions

Rate of return regulation reduces the effective cost of capital relative to that of labor for a regulated firm. This induces a bias in the capital/labor ratio chosen by the firm. When the firm produces innovations as well as its final product, rate of return regulation induces a bias towards labor augmenting innovations relative to capital augmenting innovations, except for a range of values of the elasticity of substitution in producing the final product. The typical A-J effect occurs so far as the capital/labor ratio chosen to produce the final product is concerned.

These results hold when only capital employed in producing the final product enters the rate base. If all of the firm's capital enters the rate base, then the extent of distortion in the firm's activities induced by regulation depends on the elasticity of substitution in the final product, the capital intensities of the innovative activities of the firm, and the elasticity of innovations with respect to the level of progress already achieved.

REFERENCES

Baumol, W. and Klevorick, A. "Input Choices and Rate of Return Regulation: An Overview of the Discussion." Bell Journal of Economics and Management Science, 1, No. 2 (1970): 162-190.

Samuelson, P. "A Theory of Induced Innovation Along Kennedy-Weizacker Lines." Review of Economics and Statistics, 47 (1965): 343-356.